Hyperbolic Geometry James Anderson Springer

Approximately fifty articles that were published in The Mathematical Intelligencer during its first eighteen years. The selection demonstrates the wide variety of attractive articles that have appeared over the years, ranging from general interest articles of a historical nature to lucid expositions of important current discoveries. Each article is introduced by the editors. "...The Mathematical Intelligencer publishes stylish, well-illustrated articles, rich in ideas and usually short on proofs. ...Many, but not all articles fall within the reach of the advanced undergraduate mathematics major. ... This book makes a nice addition to any undergraduate mathematics collection that does not already sport back issues of The Mathematical Intelligencer." D.V. Feldman, University of New Hamphire, CHOICE Reviews, June 2001.

This richly illustrated and clearly written undergraduate textbook captures the excitement and beauty of geometry. The approach is that of Klein in his Erlangen programme: a geometry is a space together with a set of transformations of the space. The authors explore various geometries: affine, projective, inversive, hyperbolic and elliptic. In each case they carefully explain the key results and discuss the relationships between the geometries. New features in this second edition include concise end-of-chapter summaries to aid student revision, a list of further reading and a list of special symbols. The authors have also revised many of the end-of-chapter exercises to make them more challenging and to include some interesting new results. Full solutions to the 200 problems are included in the text, while complete solutions to all of the end-of-chapter exercises are available in a new Instructors' Manual, which can be downloaded from www.cambridge.org/9781107647831.

The concept of the Euclidean simplex is important in the study of n-dimensional Euclidean geometry. This book introduces for the first time the concept of hyperbolic simplex as an important concept in n-dimensional hyperbolic geometry. Following the emergence of his gyroalgebra in 1988, the author crafted gyrolanguage, the algebraic language that sheds natural light on hyperbolic geometry and special relativity. Several authors have successfully employed the author’s gyroalgebra in their exploration for novel results. Françoise Chatelin noted in her book, and elsewhere, that the computation language of Einstein described in this book plays a universal computational role, which extends far beyond the domain of special relativity. This book will encourage researchers to use the author’s novel techniques to formulate their own results. The book provides new mathematical tools, such as hyperbolic simplexes, for the study of hyperbolic geometry in n dimensions. It also presents a new look at Einstein’s special relativity theory.

This textbook provides a unified and concise exploration of undergraduate mathematics by approaching the subject through its history. Readers will discover the rich tapestry of ideas behind familiar topics from the undergraduate curriculum, such as calculus, algebra, topology, and more. Featuring historical episodes ranging from the Ancient Greeks to Fermat and Descartes, this volume offers a glimpse into the broader context in which these ideas developed, revealing unexpected connections that make this ideal for a senior capstone course. The presentation of previous versions has been refined by omitting the less mainstream topics and inserting new connecting material, allowing instructors to cover the book in a one-semester course. This condensed edition prioritizes succinctness and cohesiveness, and there is a greater emphasis on visual clarity, featuring full color images and high quality 3D models. As in previous editions, a wide array of mathematical topics are covered, from geometry to computation; however, biographical sketches have been omitted. Mathematics and Its History: A Concise Edition is an essential resource for courses or reading programs on the history of mathematics. Knowledge of basic calculus, algebra, geometry, topology, and set theory is assumed. From reviews of previous editions: “Mathematics and Its History is a joy to read. The writing is clear, concise and inviting. The style is very different from a traditional text. I found myself picking it up to read at the expense of my usual late evening thriller or detective novel.... The author has done a wonderful job of tying together the dominant themes of undergraduate mathematics.” Richard J. Wilders, MAA, on the Third Edition "The book...is presented in a lively style without unnecessary detail. It is very stimulating and will be appreciated not only by students. Much attention is paid to problems and to the development of mathematics before the end of the nineteenth century.... This book brings to the non-specialist interested in mathematics many interesting results. It can be recommended for seminars and will be enjoyed by the broad mathematical community." European Mathematical Society, on the Second Edition

Suitable for a one-semester course in general relativity for senior undergraduates or beginning graduate students, this text clarifies the mathematical aspects of Einstein's theory of relativity without sacrificing physical understanding.

The study of 3-dimensional spaces brings together elements from several areas of mathematics. The most notable are topology and geometry, but elements of number theory and analysis also make appearances. In the past 30 years, there have been striking developments in the mathematics of 3-dimensional manifolds. This book aims to introduce undergraduate students to some of these important developments. Low-Dimensional Geometry starts at a relatively elementary level, and its early chapters can be used as a brief introduction to hyperbolic geometry. However, the ultimate goal is to describe the very recently completed geometrization program for 3-dimensional manifolds. The journey to reach this goal emphasizes examples and concrete constructions as an introduction to more general statements. This includes the tessellations associated to the process of gluing together the sides of a polygon. Bending some of these tessellations provides a natural introduction to 3-dimensional hyperbolic geometry and to the theory of kleinian groups, and it eventually leads to a discussion of the geometrization theorems for knot complements and 3-dimensional manifolds. This book is illustrated with many pictures, as the author intended to share his own enthusiasm for the beauty of some of the mathematical objects involved. However, it also emphasizes mathematical rigor and, with the exception of the most recent research breakthroughs, its constructions and statements are carefully justified.

Groups arise naturally as symmetries of geometric objects, and so groups can be used to understand geometry and topology. Conversely, one can study abstract groups by using geometric techniques and ultimately by treating groups themselves as geometric objects. This book explores these connections between group theory and geometry, introducing some of the main ideas of transformation groups, algebraic topology, and geometric group theory. The first half of the book introduces basic notions of group theory and studies symmetry groups in various geometries, including Euclidean, projective, and hyperbolic. The classification of Euclidean isometries leads to results on regular polyhedra and polytopes; the study of symmetry groups using matrices leads to Lie groups and Lie algebras. The second half of the book explores ideas from algebraic topology and geometric group theory. The fundamental group appears as yet another group associated to a geometric object and turns out to be a symmetry group using covering spaces and deck transformations. In the other direction, Cayley graphs, planar models, and fundamental domains appear as geometric objects associated to groups. The final chapter discusses groups themselves as geometric objects, including a gentle introduction to Gromov's theorem on polynomial growth and Grigorchuk's example of intermediate growth. The book is accessible to undergraduate students (and anyone else) with a background in calculus, linear algebra, and basic real analysis, including topological notions of convergence and connectedness. This book is a result of the MASS course in algebra at Penn State University in the fall semester of 2009.

Generating random networks efficiently and accurately is an important challenge for practical applications, and an interesting question for theoretical study. This book presents and discusses common methods of generating random graphs. It begins with approaches such as Exponential Random Graph Models, where the targeted probability of each network appearing in the ensemble is specified. This section also includes degree-preserving randomisation algorithms, where the aim is to generate networks with the correct number of links at each node, and care must be taken to avoid introducing a bias. Separately, it looks at growth style algorithms (e.g. preferential attachment) which aim to model a real process and then to analyse the resulting ensemble of graphs. It also covers how to generate special types of graphs including modular graphs, graphs with community structure and temporal graphs. The book is aimed at the graduate student or advanced undergraduate. It includes many worked examples and open questions making it suitable for use in teaching. Explicit pseudocode algorithms are included throughout the book to make the ideas straightforward to apply. With larger and larger datasets, it is crucial to have practical and well-understood tools. Being able to test a hypothesis against a properly specified control case is at the heart of the 'scientific method'. Hence, knowledge on how to generate controlled and unbiased random graph ensembles is vital for anybody wishing to apply network science in their research.

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A Course in Modern Geometries is designed for a junior-senior level course for mathematics majors, including those who plan to teach in secondary school. Chapter 1 presents several finite geometries in an axiomatic framework. Chapter 2 introduces Euclid's geometry and the basic ideas of non-Euclidean geometry. The synthetic approach of Chapters 1 - 2 is followed by the analytic treatment of transformations of the Euclidean plane in Chapter 3. Chapter 4 presents plane projective geometry both synthetically and analytically. The extensive use of matrix representations of groups of transformations in Chapters 3 - 4 reinforces ideas from linear algebra and serves as excellent preparation for a course in abstract algebra. Each chapter includes a list of suggested sources for applications and/or related topics.

The second edition of this book updates and expands upon a historically important collection of mathematical problems first published in the United States by Birkhäuser in 1981. These problems serve as a record of the informal discussions held by a group of mathematicians at the Scottish Café in Lwów, Poland, between the two world wars. Many of them were leaders in the development of such areas as functional and real analysis, group theory, measure and set theory, probability, and topology. Finding solutions to the problems they proposed has been ongoing since World War II, with prizes offered in many cases to those who are successful. In the 35 years since the first edition published, several more problems have been fully or partially solved, but even today many still remain unsolved and several prizes remain unclaimed. In view of this, the editor has gathered new and updated commentaries on the original 193 problems. Some problems are solved for the first time in this edition. Included again in full are transcripts of lectures given by Stanislaw Ulam, Mark Kac, Antoni Zygmund, Paul Erdös, and Andrzej Granas that provide amazing insights into the mathematical environment of Lwów before World War II and the development of The Scottish Book. Also new in this edition are a brief history of the University of Wroc?aw’s New Scottish Book, created to revive the tradition of the original, and some selected problems from it. The Scottish Book offers a unique opportunity to communicate with the people and ideas of a time and place that had an enormous influence on the development of mathematics and try their hand on the unsolved problems. Anyone in the general mathematical community with an interest in the history of modern mathematics will find this to be an insightful and fascinating read.

This book is for anyone who wishes to illustrate their mathematical ideas, which in our experience means everyone. It is organized by material, rather than by subject area, and purposefully emphasizes the process of creating things, including discussions of failures that occurred along the way. As a result, the reader can learn from the experiences of those who came before, and will be inspired to create their own illustrations. Topics illustrated within include prime numbers, fractals, the Klein bottle, Borromean rings, tilings, space-filling curves, knot theory, billiards, complex dynamics, algebraic surfaces, groups and prime ideals, the Riemann zeta function, quadratic fields, hyperbolic space, and hyperbolic 3-manifolds. Everyone who opens this book should find a type of mathematics with which they identify. Each contributor explains the mathematics behind their illustration at an accessible level, so that all readers can appreciate the beauty of both the object itself and the mathematics behind it.

This revised and greatly expanded edition of the Russian classic contains a wealth of new information about the lives of many great mathematicians and scientists, past and present. Written by a distinguished mathematician and featuring a unique mix of mathematics, physics, and history, this text combines original source material and provides careful explanations for some of the most significant discoveries in mathematics and physics. What emerges are intriguing, multifaceted biographies that will interest readers at all levels.

Richard Trudeau confronts the fundamental question of truth and its representation through mathematical models in The Non-Euclidean Revolution. First, the author analyzes geometry in its historical and philosophical setting; second, he examines a revolution every bit as significant as the Copernican revolution in astronomy and the Darwinian revolution in biology; third, on the most speculative level, he questions the possibility of absolute knowledge of the world. A portion of the book won the Pólya Prize, a distinguished award from the Mathematical Association of America.

This book illustrates how models of complex systems are built up and provides indispensable mathematical tools for studying their dynamics. This second edition includes more recent research results and many new and improved worked out examples and exercises.

This text is intended to serve as an introduction to the geometry of the action of discrete groups of Mobius transformations. The subject matter has now been studied with changing points of emphasis for over a hundred years, the most recent developments being connected with the theory of 3-manifolds: see, for example, the papers of Poincare [77] and Thurston [101]. About 1940, the now well-known (but virtually unobtainable) Fenchel-Nielsen manuscript appeared. Sadly, the manuscript never appeared in print, and this more modest text attempts to display at least some of the beautiful geo metrical ideas to be found in that manuscript, as well as some more recent material. The text has been written with the conviction that geometrical explana tions are essential for a full understanding of the material and that however simple a matrix proof might seem, a geometric proof is almost certainly more profitable. Further, wherever possible, results should be stated in a form that is invariant under conjugation, thus making the intrinsic nature of the result more apparent. Despite the fact that the subject matter is concerned with groups of isometries of hyperbolic geometry, many publications rely on Euclidean estimates and geometry. However, the recent developments have again emphasized the need for hyperbolic geometry, and I have included a comprehensive chapter on analytical (not axiomatic) hyperbolic geometry. It is hoped that this chapter will serve as a "dictionary" offormulae in plane hyperbolic geometry and as such will be of interest and use in its own right.

Appendicies A to I that are referenced by Volumes I and II in the theory of quantum torus knots (QTK). A detailed mathematical derivation of space curves is provided that links the diverse fields of superfluids, quantum mechanics, and hydrodynamics.

The subject of Kleinian groups and hyperbolic 3-manifolds is currently undergoing explosively fast development, the last few years having seen the resolution of many longstanding conjectures. This volume contains important expositions and original work by some of the main contributors on topics such as topology and geometry of 3-manifolds, curve complexes, classical Ahlfors-Bers theory, computer explorations and projective structures. Researchers in these and related areas will find much of interest here.

This introductory text provides a thoroughly modern treatment of Fuchsian groups that addresses both the classical material and recent developments in the field. A basic example of lattices in semisimple groups, Fuchsian groups have extensive connections to the theory of a single complex variable, number theory, algebraic and differential geometry, topology, Lie theory, representation theory, and group theory.

Basic Linear Algebra is a text for first year students leading from concrete examples to abstract theorems, via tutorial-type exercises. More exercises (of the kind a student may expect in examination papers) are grouped at the end of each section. The book covers the most important basics of any first course on linear algebra, explaining the algebra of matrices with applications to analytic geometry, systems of linear equations, difference equations and complex numbers. Linear equations are treated via Hermite normal forms which provides a successful and concrete explanation of the notion of linear independence. Another important highlight is the connection between linear mappings and matrices leading to the change of basis theorem which opens the door to the notion of similarity. This new and revised edition features additional exercises and coverage of Cramer's rule (omitted from the first edition). However, it is the new, extra chapter on computer assistance that will be of particular interest to readers: this will take the form of a tutorial on the use of the "LinearAlgebra" package in MAPLE 7 and will deal with all the aspects of linear algebra developed within the book.

Thorough introduction to an important area of mathematics Contains recent results Includes many exercises

Precise dynamic models of processes are required for many applications, ranging from control engineering to the natural sciences and economics. Frequently, such precise models cannot be derived using theoretical considerations alone. Therefore, they must be determined experimentally. This book treats the determination of dynamic models based on measurements taken at the process, which is known as system identification or process identification. Both offline and online methods are presented, i.e. methods that post-process the measured data as well as methods that provide models during the measurement. The book is theory-oriented and application-oriented and most methods covered have been used successfully in practical applications for many different processes. Illustrative examples in this book with real measured data range from hydraulic and electric actuators up to combustion engines. Real experimental data is also provided on the Springer webpage, allowing readers to gather their first experience with the methods presented in this book. Among others, the book covers the following subjects: determination of the non-parametric frequency response, (fast) Fourier transform, correlation analysis, parameter estimation with a focus on the method of Least Squares and modifications, identification of time-variant processes, identification in closed-loop, identification of continuous time processes, and subspace methods. Some methods for nonlinear system identification are also considered, such as the Extended Kalman filter and neural networks. The different methods are compared by using a real three-mass oscillator process, a model of a drive train. For many identification methods, hints for the practical implementation and application are provided. The book is intended to meet the needs of students and practicing engineers working in research and development, design and manufacturing.

Metric and Differential Geometry grew out of a similarly named conference held at Chern Institute of Mathematics, Tianjin and Capital Normal University, Beijing. The various contributions to this volume cover a broad range of topics in metric and differential geometry, including metric spaces, Ricci flow, Einstein manifolds, Kähler geometry, index theory, hypoelliptic Laplacian and analytic torsion. It offers the most recent advances as well as surveys the new developments. Contributors: M.T. Anderson J.-M. Bismut X. Chen X. Dai R. Harvey P. Koskela B. Lawson X. Ma R. Melrose W. Müller A. Naor J. Simons C. Sormani D. Sullivan S. Sun G. Tian K. Wildrick W. Zhang

This book comprises the Proceedings of the 12th International Congress on Mathematical Education (ICME-12), which was held at COEX in Seoul, Korea, from July 8th to 15th, 2012. ICME-12 brought together 3500 experts from 92 countries, working to understand all of the intellectual and attitudinal challenges in the subject of mathematics education as a multidisciplinary research and practice. This work aims to serve as a platform for deeper, more sensitive and more collaborative involvement of all major contributors towards educational improvement and in research on the nature of teaching and learning in mathematics education. It introduces the major activities of ICME-12 which have successfully contributed to the sustainable development of mathematics education across the world. The program provides food for thought and inspiration for practice for everyone with an interest in mathematics education and makes an essential reference for teacher educators, curriculum developers and researchers in mathematics education. The work includes the texts of the four plenary lectures and three plenary panels and reports of three survey groups, five National presentations, the abstracts of fifty one Regular lectures, reports of thirty seven Topic Study Groups and seventeen Discussion Groups.

The essential introduction to the theory and application of linear models—now in a valuable new edition Since most advanced statistical tools are generalizations of the linear model, it is neces-sary to first master the linear model in order to move forward to more advanced concepts. The linear model remains the main tool of the applied statistician and is central to the training of any statistician regardless of whether the focus is applied or theoretical. This completely revised and updated new edition successfully develops the basic theory of linear models for regression, analysis of variance, analysis of covariance, and linear mixed models. Recent advances in the methodology related to linear mixed models, generalized linear models, and the Bayesian linear model are also addressed. Linear Models in Statistics, Second Edition includes full coverage of advanced topics, such as mixed and generalized linear models, Bayesian linear models, two-way models with empty cells, geometry of least squares, vector-matrix calculus, simultaneous inference, and logistic and nonlinear regression. Algebraic, geometrical, frequentist, and Bayesian approaches to both the inference of linear models and the analysis of variance are also illustrated. Through the expansion of relevant material and the inclusion of the latest technological developments in the field, this book provides readers with the theoretical foundation to correctly interpret computer software output as well as effectively use, customize, and understand linear models. This modern Second Edition features: New chapters on Bayesian linear models as well as random and mixed linear models Expanded discussion of two-way models with empty cells Additional sections on the geometry of least squares Updated coverage of simultaneous inference The book is complemented with easy-to-read proofs, real data sets, and an extensive bibliography. A thorough review of the requisite matrix algebra has been addedfor transitional purposes, and numerous theoretical and applied problems have been incorporated with selected answers provided at the end of the book. A related Web site includes additional data sets and SAS® code for all numerical examples. Linear Model in Statistics, Second Edition is a must-have book for courses in statistics, biostatistics, and mathematics at the upper-undergraduate and graduate levels. It is also an invaluable reference for researchers who need to gain a better understanding of regression and analysis of variance.

Though mathematical ideas underpin the study of neural networks, the author presents the fundamentals without the full mathematical apparatus. All aspects of the field are tackled, including artificial neurons as models of their real counterparts; the geometry of network action in pattern space; gradient descent methods, including back-propagation; associative memory and Hopfield nets; and self-organization and feature maps. The traditionally difficult topic of adaptive resonance theory is clarified within a hierarchical description of its operation. The book also includes several real-world examples to provide a concrete focus. This should enhance its appeal to those involved in the design, construction and management of networks in commercial environments and who wish to improve their understanding of network simulator packages. As a comprehensive and highly accessible introduction to one of the most important topics in cognitive and computer science, this volume should interest a wide range of readers, both students and professionals, in cognitive science, psychology, computer science and electrical engineering.

Thoroughly updated, featuring new material on important topics such as hyperbolic geometry in higher dimensions and generalizations of hyperbolicity Includes full solutions for all exercises Successful first edition sold over 800 copies in North America

This book is an introduction to hyperbolic and differential geometry that provides material in the early chapters that can serve as a textbook for a standard upper division course on hyperbolic geometry. For that material, the students need to be familiar with calculus and linear algebra and willing to accept one advanced theorem from analysis without proof. The book goes well beyond the standard course in later chapters, and there is enough material for an honors course, or for supplementary reading. Indeed, parts of the book have been used for both kinds of courses. Even some of what is in the early chapters would surely not be nec essary for a standard course. For example, detailed proofs are given of the Jordan Curve Theorem for Polygons and of the decomposability of poly gons into triangles, These proofs are included for the sake of completeness, but the results themselves are so believable that most students should skip the proofs on a first reading. The axioms used are modern in character and more "user friendly" than the traditional ones. The familiar real number system is used as an in gredient rather than appearing as a result of the axioms. However, it should not be thought that the geometric treatment is in terms of models: this is an axiomatic approach that is just more convenient than the traditional ones.

Focussing on the geometry of hyperbolic manifolds, the aim here is to provide an exposition of some fundamental results, while being as self-contained, complete, detailed and unified as possible. Following some classical material on the hyperbolic space and the Teichmüller space, the book centers on the two fundamental results: Mostow's rigidity theorem (including a complete proof, following Gromov and Thurston) and Margulis' lemma. These then form the basis for studying Chabauty and geometric topology; a unified exposition is given of Wang's theorem and the Jorgensen-Thurston theory; and much space is devoted to the 3D case: a complete and elementary proof of the hyperbolic surgery theorem, based on the representation of three manifolds as glued ideal tetrahedra.

Microscale Diagnostic Techniques highlights the most innovative and powerful developments in microscale diagnostics. It provides a resource for scientists and researchers interested in learning about the techniques themselves, including their capabilities and limitations. The fields of Micro- and Nanotechnology have emerged over the past decade as a major focus of modern scientific and engineering research and technology. Driven by advances in microfabrication, the investigation, manipulation and engineering of systems characterized by micrometer and, more recently, nanometer scales have become commonplace throughout all technical disciplines. With these developments, an entirely new collection of experimental techniques has been developed to explore and characterize such systems.

Although it arose from purely theoretical considerations of the underlying axioms of geometry, the work of Einstein and Dirac has demonstrated that hyperbolic geometry is a fundamental aspect of modern physics. In this book, the rich geometry of the hyperbolic plane is studied in detail, leading to the focal point of the book, Poincare's polygon theorem and the relationship between hyperbolic geometries and discrete groups of isometries. Hyperbolic 3-space is also discussed, and the directions that current research in this field is taking are sketched. This will be an excellent introduction to hyperbolic geometry for students new to the subject, and for experts in other fields.

This book provides an introduction to hyperbolic geometry in dimension three, with motivation and applications arising from knot theory. Hyperbolic geometry was first used as a tool to study knots by Riley and then Thurston in the 1970s. By the 1980s, combining work of Mostow and Prasad with Gordon and Luecke, it was known that a hyperbolic structure on a knot complement in the 3-sphere gives a complete knot invariant. However, it remains a difficult problem to relate the hyperbolic geometry of a knot to other invariants arising from knot theory. In particular, it is difficult to determine hyperbolic geometric information from a knot diagram, which is classically used to describe a knot. This textbook provides background on these problems, and tools to determine hyperbolic information on knots. It also includes results and state-of-the art techniques on hyperbolic geometry and knot theory to date. The book was written to be interactive, with many examples and exercises. Some important results are left to guided exercises. The level is appropriate for graduate students with a basic background in algebraic topology, particularly fundamental groups and covering spaces. Some experience with some differential topology and Riemannian geometry will also be helpful.

The Springer Handbook of Spacetime is dedicated to the ground-breaking paradigm shifts embodied in the two relativity theories, and describes in detail the profound reshaping of physical sciences they ushered in. It includes in a single volume chapters on foundations, on the underlying mathematics, on physical and astrophysical implications, experimental evidence and cosmological predictions, as well as chapters on efforts to unify general relativity and quantum physics. The Handbook can be used as a desk reference by researchers in a wide variety of fields, not only by specialists in relativity but also by researchers in related areas that either grew out of, or are deeply influenced by, the two relativity theories: cosmology, astronomy and astrophysics, high energy physics, quantum field theory, mathematics, and philosophy of science. It should also serve as a valuable resource for graduate students and young researchers entering these areas, and for instructors who teach courses on these subjects. The Handbook is divided into six parts. Part A: Introduction to Spacetime Structure. Part B: Foundational Issues. Part C: Spacetime Structure and Mathematics. Part D: Confronting Relativity theories with observations. Part E: General relativity and the universe. Part F: Spacetime beyond Einstein.

Drawing on many years'experience of teaching discrete mathem atics to students of all levels, Anderson introduces such as pects as enumeration, graph theory and configurations or arr angements. Starting with an introduction to counting and rel ated problems, he moves on to the basic ideas of graph theor y with particular emphasis on trees and planar graphs. He de scribes the inclusion-exclusion principle followed by partit ions of sets which in turn leads to a study of Stirling and Bell numbers. Then follows a treatment of Hamiltonian cycles, Eulerian circuits in graphs, and Latin squares as well as proof of Hall's theorem. He concludes with the constructions of schedules and a brief introduction to block designs. Each chapter is backed by a number of examples, with straightforw ard applications of ideas and more challenging problems.

This book is an exposition of the theoretical foundations of hyperbolic manifolds. It is intended to be used both as a textbook and as a reference. Particular emphasis has been placed on readability and completeness of ar gument. The treatment of the material is for the most part elementary and self-contained. The reader is assumed to have a basic knowledge of algebra and topology at the first-year graduate level of an American university. The book is divided into three parts. The first part, consisting of Chap ters 1-7, is concerned with hyperbolic geometry and basic properties of discrete groups of isometries of hyperbolic space. The main results are the existence theorem for discrete reflection groups, the Bieberbach theorems, and Selberg's lemma. The second part, consisting of Chapters 8-12, is de voted to the theory of hyperbolic manifolds. The main results are Mostow's rigidity theorem and the determination of the structure of geometrically finite hyperbolic manifolds. The third part, consisting of Chapter 13, in tegrates the first two parts in a development of the theory of hyperbolic orbifolds. The main results are the construction of the universal orbifold covering space and Poincare's fundamental polyhedron theorem.

This textbook offers a geometric perspective on special relativity, bridging Euclidean space, hyperbolic space, and Einstein’s spacetime in one accessible, self-contained volume. Using tools tailored to undergraduates, the author explores Euclidean and non-Euclidean geometries, gradually building from intuitive to abstract spaces. By the end, readers will have encountered a range of topics, from isometries to the Lorentz–Minkowski plane, building an understanding of how geometry can be used to model special relativity. Beginning with intuitive spaces, such as the Euclidean plane and the sphere, a structure theorem for isometries is introduced that serves as a foundation for increasingly sophisticated topics, such as the hyperbolic plane and the Lorentz–Minkowski plane. By gradually introducing tools throughout, the author offers readers an accessible pathway to visualizing increasingly abstract geometric concepts. Numerous exercises are also included with selected solutions provided. Geometry: from Isometries to Special Relativity offers a unique approach to non-Euclidean geometries, culminating in a mathematical model for special relativity. The focus on isometries offers undergraduates an accessible progression from the intuitive to abstract; instructors will appreciate the complete instructor solutions manual available online. A background in elementary calculus is assumed.

Completely revised text applies spectral methods to boundary value, eigenvalue, and time-dependent problems, but also covers cardinal functions, matrix-solving methods, coordinate transformations, much more. Includes 7 appendices and over 160 text figures.

Standard approaches to understanding swarms rely on inspiration from biology and are generally covered by the term “biomimetics”. This book focuses on a different, complementary inspiration, namely physics. The editors have introduced the term 'physicomimetics' to refer to physics-based swarm approaches, which offer two advantages. First, they capture the notion that “nature is lazy', meaning that physics-based systems always perform the minimal amount of work necessary, which is an especially important advantage in swarm robotics. Second, physics is the most predictive science, and can reduce complex systems to simple concepts and equations that codify emergent behavior and help us to design and understand swarms. The editors consolidated over a decade of work on swarm intelligence and swarm robotics, organizing the book into 19 chapters as follows. Part I introduces the concept of swarms and offers the reader a physics tutorial; Part II deals with applications of physicomimetics, in order of increased complexity; Part III examines the hardware requirements of the presented algorithms and demonstrates real robot implementations; Part IV demonstrates how the theory can be used to design swarms from first principles and provides a novel algorithm that handles changing environments; finally, Part V shows that physicomimetics can be used for function optimization, moving the reader from issues of swarm robotics to swarm intelligence. The text is supported with a downloadable package containing simulation code and videos of working robots. This book is suitable for talented high school and undergraduate students, as well as researchers and graduate students in the areas of artificial intelligence and robotics.